Conditional damped random surface velocity model of turbulent jet breakup

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Goal: Model turbulent jet breakup at low ambient densities

- Liquid jet in low density quiescent gaseous environment
- Jet is statistically steady
- Jet is turbulent at nozzle exit (No transition)
- Newtonian fluid, low Mach number

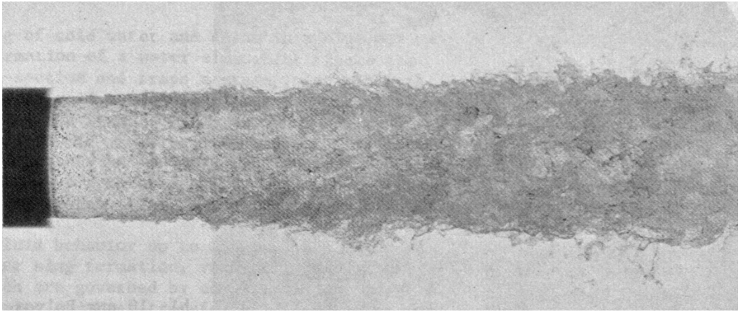
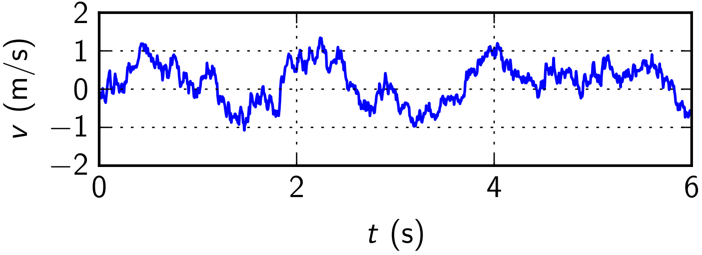


Figure: Pipe jet from Hoyt and Taylor (1980, fig. 1a). $\mathrm{Re}_{l0} \approx 5 \cdot 10^5$.

Basic ideas of model

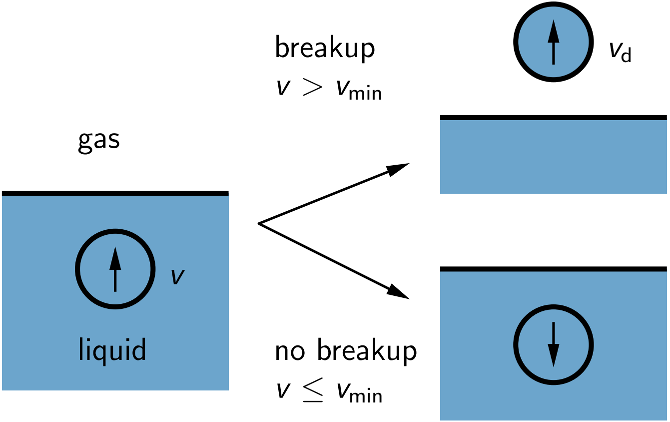
- Phenomenological model framework — I hope I am asking the right questions as I'm providing fully right answers.
- Model based around turbulence as a random process. Averages are computed directly, so CDRSV theory is an improvement over scaling estimates, e.g., Faeth group theory.
- Averages are *conditional*, because the definitions of the quantities of interest imply droplets are formed.



Ex.: mean droplet radial velocity *without damping* (1/3)

Jet breakup models often *assume* $\langle v_d \rangle \propto v'$. CDRSV theory can easily *mathematically derive* this result and predict the constant.

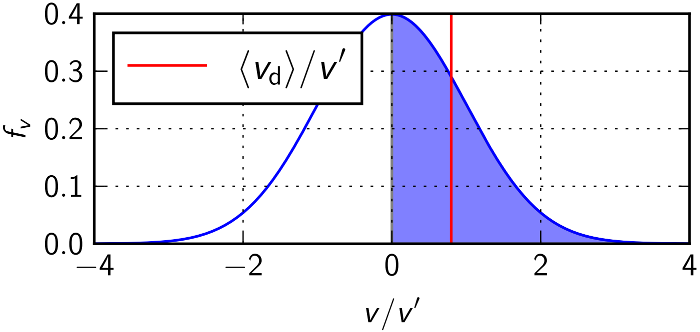
An eddy of radial velocity v is approaching the free surface.



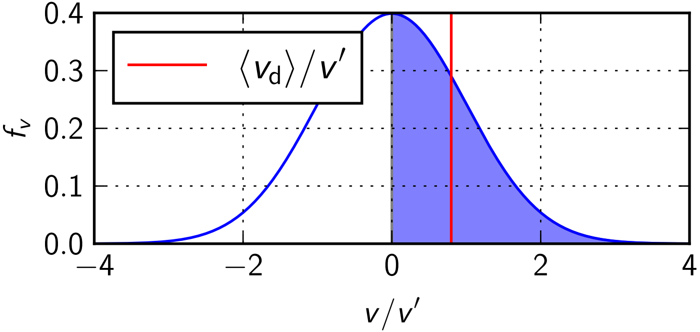
Ex.: mean droplet radial velocity *without damping* (2/3)

In this example, the free surface presents no obstacle to the eddy. So $v_\mathrm{d} = v$. (No damping.)

A droplet is formed if $v_\mathrm{d} > 0$, so if $v > v_\mathrm{min} = 0$ (in this case). I abbreviate this condition as DF (droplet formation).



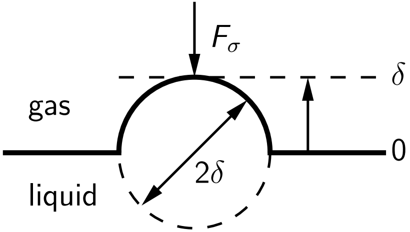
Ex.: mean droplet radial velocity *without damping* (3/3)



$$\langle v_\mathrm{d} \rangle = \langle v_\mathrm{d} \mid \mathrm{DF} \rangle = \langle v \mid \mathrm{DF} \rangle = \int_{v_\mathrm{min}}^{\infty} v \cdot \underbrace{\left(\frac{f(v)}{\int_{v_\mathrm{min}}^{\infty} f(v)\,\mathrm{d}v} \right)}_{\text{conditional density}} \mathrm{d}v$$

$$= \int_0^\infty \frac{2v}{\sqrt{2\pi v'^2}} \exp\left(-\frac{v^2}{2v'^2} \right) \mathrm{d}v = \sqrt{\frac{2}{\pi}}\, v'$$

Droplet radial velocity model



$$v_\mathrm{d} = v\sqrt{1 - \frac{12 C_\mathrm{lig}^2 C_F}{C_\forall}\frac{\sigma}{\rho_\mathrm{l} v^2 \ell}} = v\sqrt{1 - \frac{\mathrm{We}_\mathrm{T,crit}}{\mathrm{We}_\mathrm{T}}}$$

leading to the minimum (Hinze) scales if an inertial range spectrum is used for the eddy sizes:

$$v_\sigma \equiv \underbrace{\left(\frac{\mathrm{We}_\mathrm{T,crit}}{2\pi}\right)^{1/5}}_{C_{v_\sigma}} \left(\frac{\sigma\varepsilon}{\rho_\mathrm{l}}\right)^{1/5}$$

Sauter mean diameter (D_{32}) theory (1/2)

Wu, Tseng, and Faeth (1992, p. 312) assume that $D_{32} \propto \langle \ell \mid \mathrm{DF} \rangle$.

I instead apply an energy balance:

$$\sigma \frac{SA}{\forall} = \tfrac{1}{2}\rho_\mathrm{l} \left(v^2 - v_\mathrm{d}^2\right).$$

Averaging:

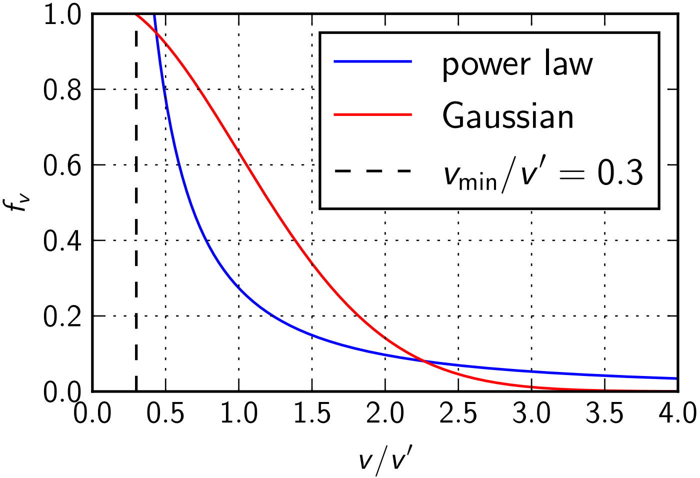
$$\sigma \left\langle \frac{SA}{\forall} \right\rangle = \tfrac{1}{2}\rho_\mathrm{l} \left\langle v^2 - v_\mathrm{d}^2 \;\middle|\; \mathrm{DF} \right\rangle = \tfrac{1}{2}\rho_\mathrm{l} \mathrm{We}_{\mathrm{T,crit}} \left\langle \frac{v^2}{\mathrm{We}_\mathrm{T}} \;\middle|\; \mathrm{DF} \right\rangle,$$

which returns (using $\langle SA/\forall \rangle \approx \langle SA \rangle / \langle \forall \rangle = 6/D_{32}$):

$$D_{32} = \frac{12}{\mathrm{We}_{\mathrm{T,crit}}} \left\langle \ell^{-1} \;\middle|\; \mathrm{DF} \right\rangle^{-1}.$$

(A *harmonic* mean, not arithmetic!)

Velocity fluctuation probability density functions



$$f_v(v) = \frac{\alpha - 1}{v_{\min}} \left(\frac{v}{v_{\min}} \right)^{-\alpha} \longrightarrow f_v(v) \propto v^{-\alpha}$$

Sauter mean diameter (D_{32}) theory (2/2)

Power law velocity probability density function: $f_v(v) \propto v^{-\alpha}$.

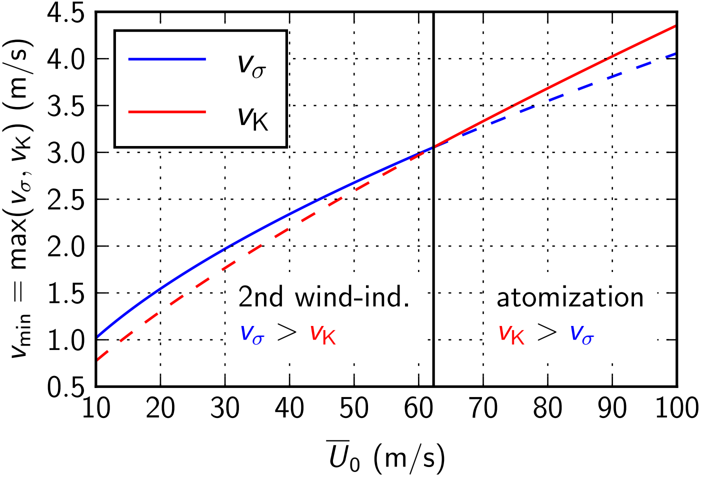
Using the inertial range spectrum to relate eddy velocities to eddy length scales returns:

$$\frac{D_{32}}{d_0} = \frac{24\pi}{\mathrm{We}_{\mathrm{T,crit}}} \frac{\left\langle v^{-3} \mid v > v_{\min} \right\rangle}{C_{\mathrm{K}}^{1/2} \varepsilon} = C_{D_{32}} \overline{\mathrm{Tu}_0}^{-6/5} \mathrm{We}_{\mathrm{l0}}^{-3/5} \left(\frac{\Lambda_0}{d_0} \right)^{2/5}.$$

Compare against empirical regression (initial droplet diameter data from Faeth group):

$$\frac{D_{32}}{d_0} = 0.8082 \left(\overline{\mathrm{Tu}_0}^2 \mathrm{We}_{\mathrm{l0}} \right)^{-0.6988}.$$

Regime transition theory (1/2)



v_σ — Hinze scale
(surface tension controlled)

v_K — Kolmogorov scale
(viscosity controlled)

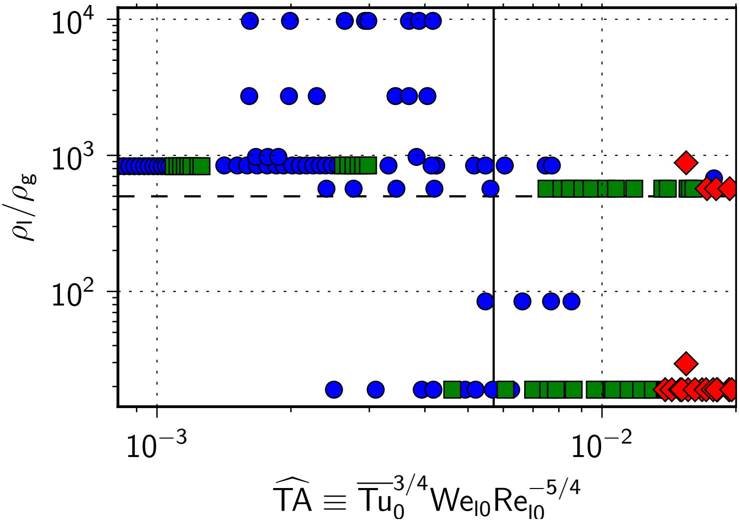
Regime transition theory (2/2)

- Equating the v_σ and v_K and rearranging leads to:

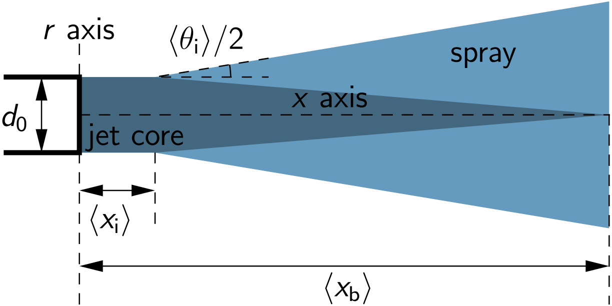
$$\mathsf{TA}_\mathsf{crit} = C_\varepsilon^{1/4} \left(\frac{3}{2}\right)^{3/8} \left[\left(\overline{\mathsf{Tu}}_0^3 \frac{d_0}{\Lambda_0} \right)^{1/4} \mathsf{We}_{\mathsf{l0}} \mathsf{Re}_{\mathsf{l0}}^{-5/4} \right]_\mathsf{crit}.$$

- For simplicity I'll define $\widehat{\mathsf{TA}} \equiv \overline{\mathsf{Tu}}_0^{3/4} \mathsf{We}_{\mathsf{l0}} \mathsf{Re}_{\mathsf{l0}}^{-5/4}$.

Regime transition data comparison



Jet quantities of interest



- d_0 — nozzle diameter
- D_{32} — Sauter mean diameter
- $\langle v_d \rangle$ — mean radial droplet velocity (only simplified example in talk)
- $\langle x_i \rangle$ — average breakup onset location (not in talk)
- $\langle x_b \rangle$ — average breakup length (briefly in talk)
- $\langle \theta_i \rangle$ — average spray angle (not in talk)

CDRSV theory summary

- Ensemble averages in turbulent breakup often are conditioned on droplet formation.
- Explicit computation of averages can be more insightful than scaling arguments.
- The change in the minimum scale from the Hinze scale to Kolmogorov scale may partially explain the turbulent atomization regime at low atmospheric densities.
- CDRSV theory suggests there is a Reynolds number dependence in the atomization regime but not the second wind-induced regime.
- **Current CDRSV theory is a work in progress.**

Questions?

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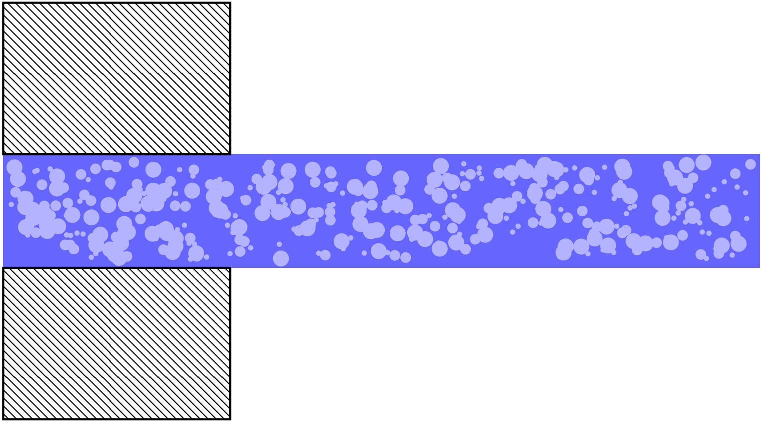
Paper:

CDRSV model of turbulent jet breakup
Preprint DOI: 10.17605/OSF.IO/35U7G

Turbulent vs. shear mechanisms of breakup



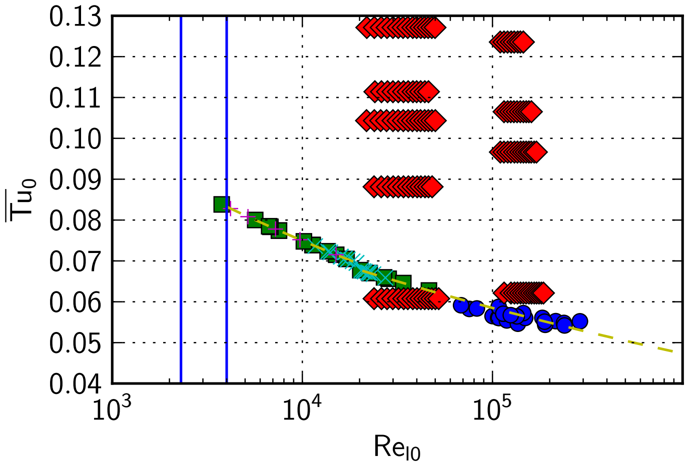
Turbulence is accepted to be the primary cause of breakup at low atmospheric densities. This turbulence is developed upstream and in the nozzle. Shear (e.g., Kelvin-Helmholtz instability) has a reduced role in this case.

Validation data compilation

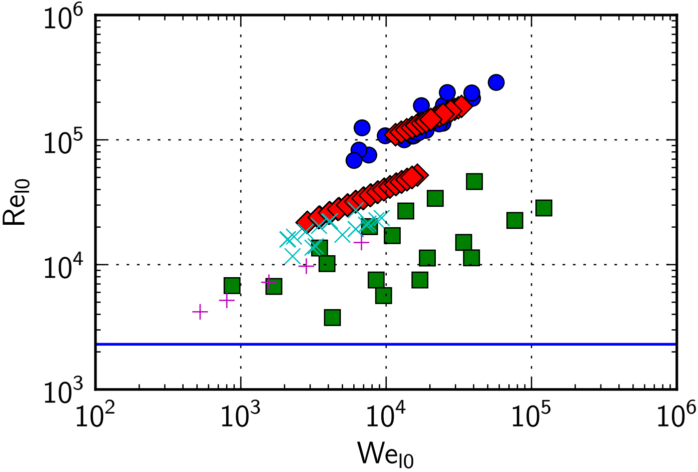
- Using data compiled from many fully developed "pipe jet" studies because they are common and well characterized.
- Problems with previous experiments:
  - little or unknown variation in $\overline{\mathrm{Tu}}_0$ (solved by rough pipes)
  - apples to oranges comparisons: need nozzle standardization to ensure that other variables (e.g., the integral scales, velocity profile, anisotropy) remain consistent
- Can estimate turbulence intensity from friction factor (idea from Skrebkov (1966), but regression is my own):

$$\overline{\mathrm{Tu}}_0 = \frac{\sqrt{\frac{2}{3}\overline{k}_0}}{\overline{U}_0} = \frac{\sqrt{\overline{{u'_0}^2}}}{\overline{U}_0} = 0.36552 f^{0.45867}$$

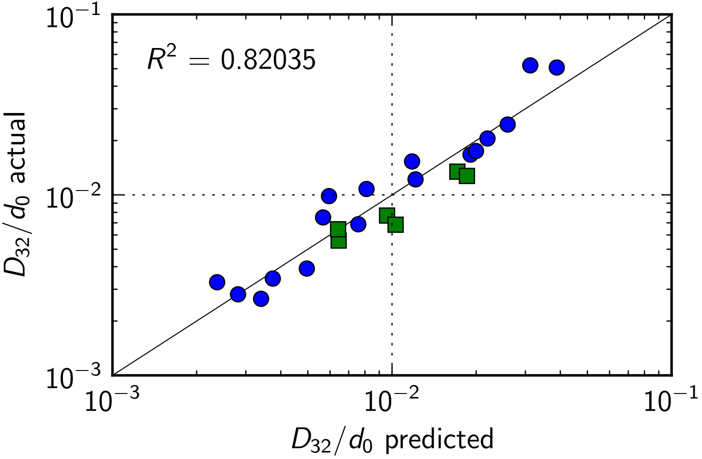
Confounding example 1



Confounding example 2



Sauter mean diameter (D_{32}) data comparison (regression)



Mean droplet radial velocity ($\langle v_\mathrm{d} \rangle$) theory

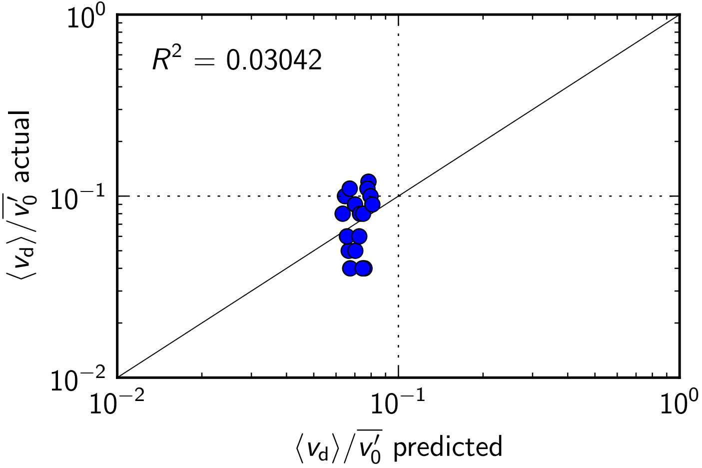
Apply a similar procedure for mean droplet radial velocity:

$$\frac{\langle v_\mathrm{d} \rangle}{\overline{v_0'}} \approx \frac{\langle v \mid \mathrm{DF} \rangle}{\overline{v_0'}} \left\langle 1 - \mathrm{We}_{\mathrm{T,crit}} \frac{\sigma}{\rho_\mathrm{l} v^2 \ell} \;\middle|\; \mathrm{DF} \right\rangle^{1/2}$$
$$= C_{v_\mathrm{d}} \overline{\mathrm{Tu}_0}^{-2/5} \left(\mathrm{We}_{\mathrm{l}0} \frac{\Lambda_0}{d_0} \right)^{-1/5}.$$

Compare against empirical regression:

$$\frac{\langle v_\mathrm{d} \rangle}{\overline{v_0'}} = 0.0487 \left(\overline{\mathrm{Tu}_0}^2 \mathrm{We}_{\mathrm{l}0} \right)^{0.0607}.$$

Mean droplet radial velocity ($\langle v_\mathrm{d} \rangle$) data comparison (regression)



Breakup onset location (x_i) theory

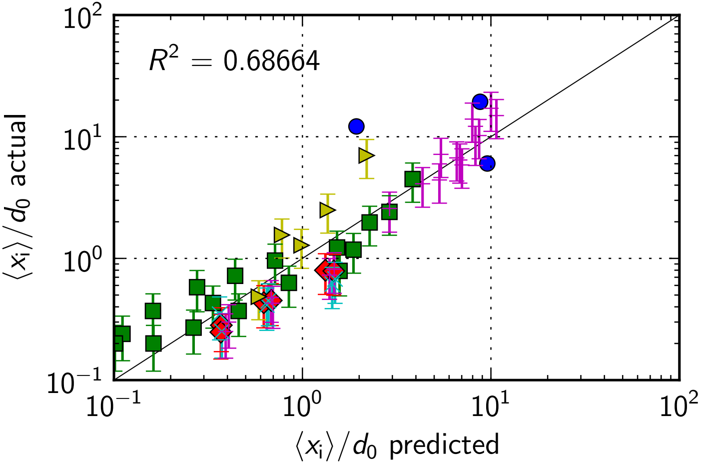
Assume that $\langle x_\mathrm{i} \rangle \approx \overline{U}_0 \langle t_\mathrm{b} \rangle$ and note that to second order $t_\mathrm{b} = C_\mathrm{lig} \ell / v$. Now we have $\langle t_\mathrm{b} \rangle \propto \left\langle \ell / v \;\middle|\; \mathrm{DF} \right\rangle$, which by hypothesis is not affected by the small scales much, so the conditioning has no effect. Then dimensional analysis suggests:

$$\frac{\langle x_\mathrm{i} \rangle}{d_0} = \frac{\overline{U}_0 \langle t_\mathrm{b} \rangle}{d_0} = \frac{C_\mathrm{lig} \overline{U}_0}{d_0} \left\langle \frac{\ell}{v} \;\middle|\; \mathrm{DF} \right\rangle = C_{x_\mathrm{i}} \overline{\mathrm{Tu}_0}^{-3} \mathrm{We}_{\mathrm{l0}}^{-1}.$$

Compare against empirical regression:

$$\frac{\langle x_\mathrm{i} \rangle}{d_0} = 16.0298 (\mathrm{Tu}_0^3 \mathrm{We}_{\mathrm{l0}})^{-0.9567}$$

Breakup onset location (x_i) data comparison (regression)



Breakup length ($\langle x_{\mathrm{b}} \rangle$) theory (1/2)

Average mass flux:

$$\langle \dot{m}'' \rangle \equiv C_m \left\langle \frac{1}{\ell}\frac{1}{\ell}\frac{v}{\ell}\rho_{\mathrm{l}}\ell^3 \;\middle|\; \mathrm{DF} \right\rangle = C_m \rho_{\mathrm{l}} \langle v \mid \mathrm{DF} \rangle.$$

Jet core mass differential equation:

$$\frac{\mathrm{d}(\rho_{\mathrm{l}} A(x) \overline{U}_0)}{\mathrm{d}x} = -P(x)\dot{m}'' \quad \longrightarrow \quad \frac{\mathrm{d}\langle d_{\mathrm{j}} \rangle}{\mathrm{d}x} = -\frac{2\langle \dot{m}'' \rangle}{\rho_{\mathrm{l}} \overline{U}_0}.$$

Solution:

$$\frac{\langle x_{\mathrm{b}} \rangle}{d_0} = \frac{\rho_{\mathrm{l}} \overline{U}_0}{2\langle \dot{m}'' \rangle} = C_{x_{\mathrm{b}}} \overline{\mathrm{Tu}_0}^{-3/5} \left(\mathrm{We}_{\mathrm{l}0} \frac{\Lambda_0}{d_0} \right)^{1/5}.$$

Breakup length ($\langle x_b \rangle$) theory (2/2)

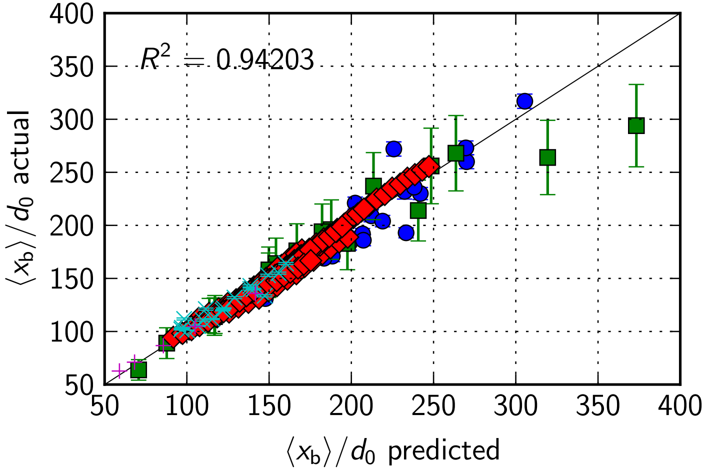
Theory:

$$\frac{\langle x_\mathrm{b} \rangle}{d_0} = C_{x_\mathrm{b}} \overline{\mathrm{Tu}_0}^{-3/5} \left(\mathrm{We}_{\mathrm{l}0} \frac{\Lambda_0}{d_0} \right)^{1/5}.$$

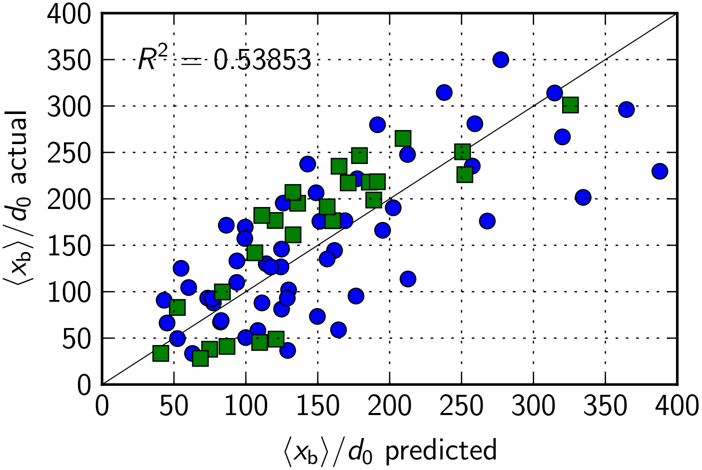
Compare against empirical regression:

$$\frac{\langle x_\mathrm{b} \rangle}{d_0} = 3.8911 \overline{\mathrm{Tu}_0}^{-0.2685} \mathrm{We}_{\mathrm{l}0}^{0.3273}.$$

Breakup length ($\langle x_b \rangle$) data comparison (regression)



Breakup length ($\langle x_b \rangle$) cross validation (regression)



Breakup length regimes

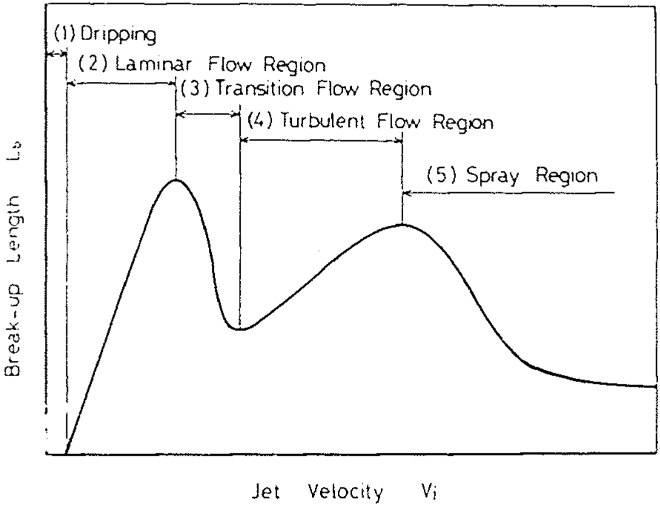


Figure: Breakup length trends from Hiroyasu, Shimizu, and Arai (1982).

Spray angle ($\langle\theta_{\rm i}\rangle$) theory

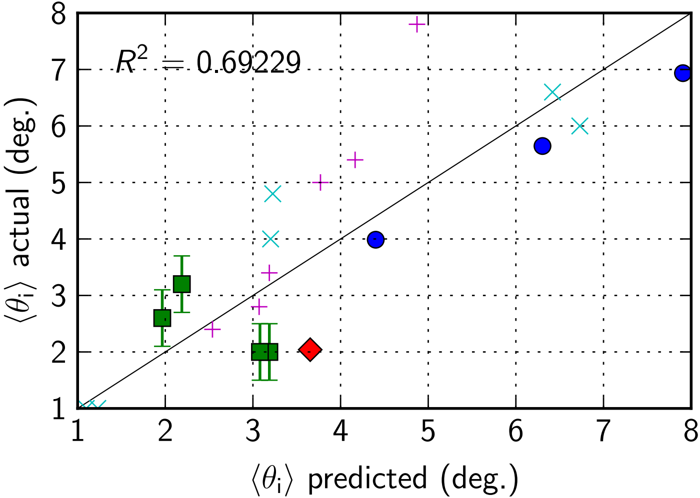
Theory:

$$\tan\left(\frac{\langle\theta\rangle}{2}\right) \approx \left\langle \tan\frac{\theta}{2} \right\rangle \equiv \left\langle \frac{v_{\rm d}}{u_{\rm d}} \right\rangle \approx \frac{\langle v_{\rm d}\rangle}{\langle u_{\rm d}\rangle} = C_{v_{\rm d}} \overline{\mathrm{Tu}_0}^{3/5} \left(\mathrm{We}_{\rm l0} \frac{\Lambda_0}{d_0}\right)^{-1/5}.$$

Compare against empirical regression:

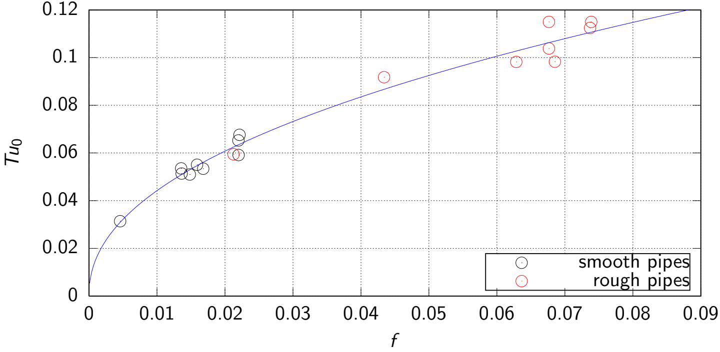
$$\tan\left(\frac{\langle\theta\rangle}{2}\right) = 0.003113 \overline{\mathrm{Tu}_0}^{0.8730} \mathrm{We}_{\rm l0}^{0.4294}.$$

Spray angle ($\langle\theta_i\rangle$) data comparison (regression)



Fully developed pipe $\overline{\mathrm{Tu}}$ regression

$$\overline{\mathrm{Tu}}_0 = \frac{\sqrt{\frac{2}{3}\overline{k}_0}}{\overline{U}_0} = \frac{\sqrt{\overline{u_0'^2}}}{\overline{U}_0} = 0.36552 f^{0.45867}$$



Is the minimum scale zero? (In reply to Marios Soteriou's keynote speech)

- Eddy vs. surface perturbation vs. droplet sizes.
- There may be a minimum for primary breakup but not secondary breakup.
- The distribution may be well approximated by a minimum at a certain point, below which the frequency of occurrence is small.
- CDRSV theory at the moment assumes one droplet is formed per eddy event. I have done some work to generalize the theory beyond this, which would suggest that arbitrarily small droplets can be formed from eddy velocities which are not commensurately small. So the minimum velocity idea can still be valid even with a minimum droplet diameter of zero.

References I

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