Estimating turbulent kinetic energy and dissipation with internal flow loss coefficients

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Goal: Estimate nozzle outlet turbulent kinetic energy

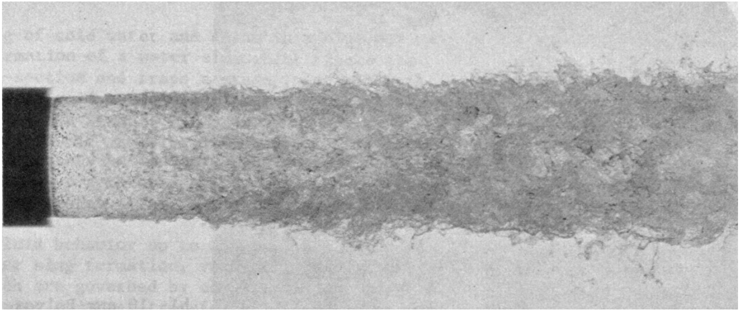
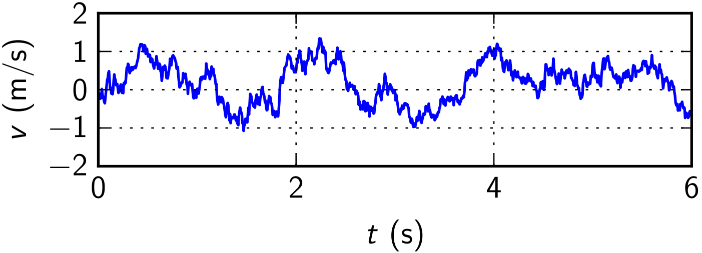


Figure: Pipe jet from Hoyt and Taylor (1980, fig. 1a). $\mathrm{Re}_{l0} \approx 5 \cdot 10^5$.

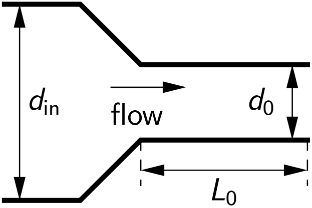
See my other ICLASS 2018 paper for details of turbulent breakup:
CDRSV model of turbulent jet breakup
Preprint DOI: 10.17605/OSF.IO/35U7G

Background: Basic turbulence terminology



- u' — RMS velocity — the standard deviation of the velocity
- $k \equiv \frac{1}{2}(u'^2 + v'^2 + w'^2)$ — turbulent kinetic energy
- ε — turbulence dissipation rate
- Bars — plane averages. Brackets — ensemble or time averages.
- $\mathrm{Tu} \equiv u'/\overline{U}$ — turbulence intensity

Background: Huh's model for nozzle turbulence (1/4)



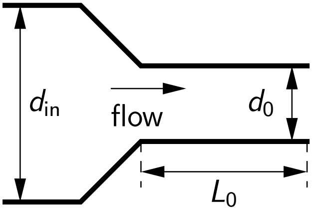
Huh and Gosman (1991) and Huh, Lee, and Koo (1998) developed the following model for nozzle turbulent kinetic energy:

$$\overline{k}_0 = \frac{\overline{U}_0^2}{8 L_0 / d_0} \left[\frac{1}{C_{\text{d}}^2} - \zeta_{\text{c}} - \left(1 - c^{-2} \right) \right]$$

Note: 0 subscript is at nozzle exit, $c = (d_{\text{in}}/d_0)^2$, ζ_{c} is the contraction loss coefficient

But this model fails several sanity checks.

Background: Huh's model for nozzle turbulence (2/4)



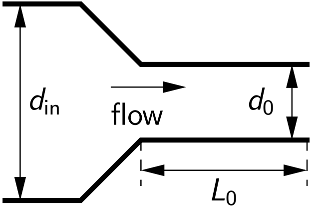
$$\overline{k}_0 = \frac{\overline{U}_0^2}{8 L_0 / d_0} \left[\frac{1}{C_d^2} - \zeta_c - \left(1 - c^{-2} \right) \right]$$

Sanity check: Has the model been experimentally validated?

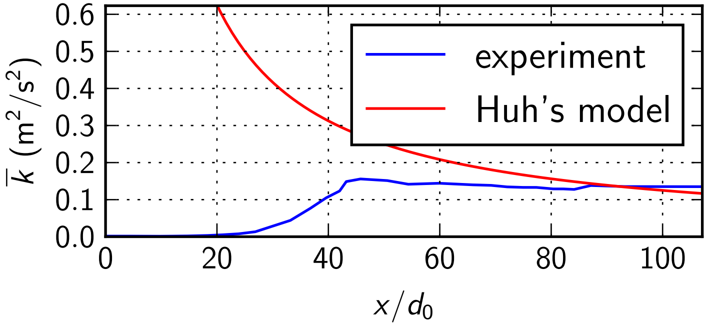
No, not directly. The nozzle model combined with the breakup model was validated against spray angle data. There was no direct, unambiguous comparison of the nozzle model in isolation against nozzle turbulence data.

I will compare against some limited data on flow development in pipes, not nozzles.

Background: Huh's model for nozzle turbulence (3/4)



$$\overline{k}_0 = \frac{\overline{U}_0^2}{8 L_0 / d_0} \left[\frac{1}{C_d^2} - \zeta_c - \left(1 - c^{-2}\right) \right]$$



Background: Huh's model for nozzle turbulence (4/4)

These problems come from an error in the derivation of the equation. Specifically:

$$\rho_l {u'}^2 \pi d_0 L_0 = \Delta p_{\text{nozzle}} \frac{\pi d_0^2}{4}.$$

This assumes that the only source or sink of turbulence is the nozzle walls, which is false — the model neglects the inlet and contraction.

It also confuses the Reynolds shear stress $\langle uv \rangle$ with the turbulent kinetic energy k. The two are not the same and you can not easily convert between the two.

Worse, the model uses a force balance at the wall, which is invalid because both $\langle uv \rangle$ and k are zero at smooth walls.

Linking turbulent kinetic energy and loss coefficients (1/3)

While Huh's model is inaccurate, the basic idea that you can link turbulent kinetic energy and internal flow loss coefficients is valid and has a long history.

To connect the two, I derived the turbulent (RANS) version of the Bernoulli theorem (for a streamtube):

$$0 = \dot{m} \left[\frac{\alpha \overline{U}^2}{2} + \frac{\overline{P}}{\rho} + \overline{k} \right]_1^2 + \rho \int_{\mathrm{CS}} \int_{\mathrm{s}} \left[\varepsilon_{\mathrm{m}} + \varepsilon - \nu \frac{\partial}{\partial x_j} \left(U_i \frac{\partial U_i}{\partial x_j} \right) + \frac{1}{2} \frac{\partial \langle u_j u_i u_i \rangle}{\partial x_j} - \nu \frac{\partial^2 k}{\partial x_i \partial x_i} + \frac{\partial U_i \langle u_i u_j \rangle}{\partial x_j} - \frac{1}{\rho} \frac{\partial \langle u_i p \rangle}{\partial x_i} \right] \mathrm{d}\forall$$

The volume integral term is awful!

Linking turbulent kinetic energy and loss coefficients (2/3)

If we assume that

$$\rho \int_{CS} \int_{s} \left[-\nu \frac{\partial}{\partial x_j} \left(U_i \frac{\partial U_i}{\partial x_j} \right) + \frac{1}{2} \frac{\partial \langle u_j u_i u_i \rangle}{\partial x_j} - \nu \frac{\partial^2 k}{\partial x_i \partial x_i} + \frac{\partial U_i \langle u_i u_j \rangle}{\partial x_j} - \frac{1}{\rho} \frac{\partial \langle u_i p \rangle}{\partial x_i} \right] \mathrm{d}\forall = 0$$

Then we have

$$0 = \left[\frac{\alpha \overline{U}^2}{2} + \frac{\overline{P}}{\rho} + \overline{k} \right]_1^2 + \frac{\rho}{\dot{m}} \int_{\forall} (\varepsilon_{\mathrm{m}} + \varepsilon) \, \mathrm{d}\forall$$

Linking turbulent kinetic energy and loss coefficients (3/3)

The deviation from the typical Bernoulli equation is generally called "loss":

$$\text{loss} = \sum \zeta \cdot \tfrac{1}{2}\overline{U}^2 = \Delta\overline{k} + \frac{\rho}{\dot{m}} \int_{\forall} (\varepsilon_\mathrm{m} + \varepsilon)\, \mathrm{d}\forall.$$

The energy "loss" is decomposed into three components

1. $\Delta\overline{k}$ — mean flow energy converted into turbulent kinetic energy
2. $(\rho/\dot{m}) \int_{\forall} \varepsilon_\mathrm{m}\, \mathrm{d}\forall$ — energy dissipated by the mean flow
3. $(\rho/\dot{m}) \int_{\forall} \varepsilon\, \mathrm{d}\forall$ — energy dissipated by turbulence

Fully developed pipe $\overline{\mathrm{Tu}}$ theory

Using the dissipation model $\varepsilon = C_\varepsilon k^{3/2}/\Lambda$ and assuming the previously neglected terms are zero, you can derive the following.

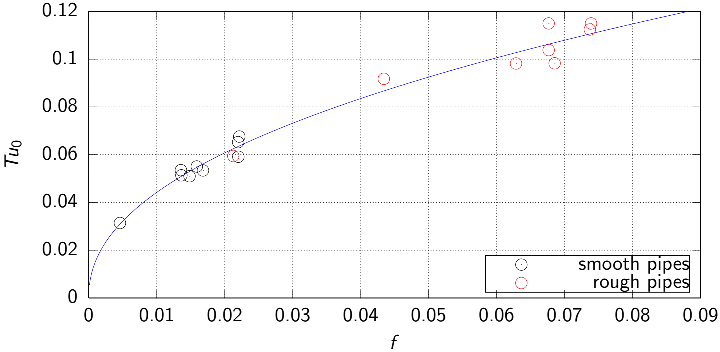
$$\overline{k}_{\mathrm{FD}} = \overline{U}^2 \left(\frac{f}{2C_\varepsilon} \right)^{2/3}$$

$$\overline{\mathrm{Tu}}_{\mathrm{FD}} = \sqrt{\frac{2}{3}} \left(\frac{f}{2C_\varepsilon} \right)^{1/3}$$

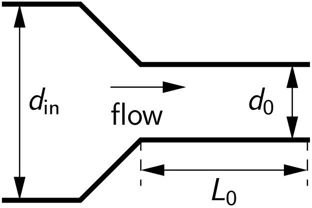
Fully developed pipe $\overline{\mathrm{Tu}}$ regression

$$\overline{\mathrm{Tu}}_{\mathrm{FD}} = \frac{\sqrt{\frac{2}{3}\overline{k}_{\mathrm{FD}}}}{\overline{U}_0} = \frac{\sqrt{\overline{u_0'^2}}}{\overline{U}_0} = 0.36552 f^{0.45867}$$

vs. theory: $\overline{\mathrm{Tu}}_0 \propto f^{1/3}$



Nozzle turbulence model (1/4)

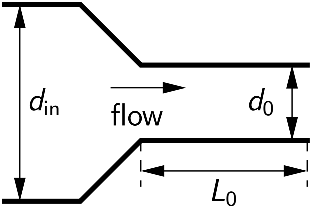


Apply rapid distortion theory (RDT) to the contraction (note that $c = (d_{in}/d_0)^2$ and $b \approx -1/8$ for pipe flows):

$$\overline{\mathrm{Tu}_c^2} = \frac{3}{4}\left(\frac{\overline{\mathrm{Tu}_{in}}}{c}\right)^2 \left[\frac{\left(\frac{1}{3} - 2b\right)\left[\ln(4c^3) - 1\right]}{c^2} + 2\left(b + \frac{1}{3}\right)c\right]$$

Plug this into the equation on the next slide...

Nozzle turbulence model (2/4)



Linearize the differential equation for the orifice to find:

$$\overline{\mathrm{Tu}}_0^2 = \overline{\mathrm{Tu}}_{\mathrm{FD}}^2 + \left(\overline{\mathrm{Tu}}_{\mathrm{c}}^2 - \overline{\mathrm{Tu}}_{\mathrm{FD}}^2\right) \exp\left(-\frac{3\overline{\mathrm{Tu}}_{\mathrm{FD}}^2}{f}\frac{L_0}{d_0}\right)$$

This equation has the right trends, but based on available data appears to become fully developed too soon. Adjusting the 3 is one empirical way to correct this.

Nozzle turbulence model (3/4)

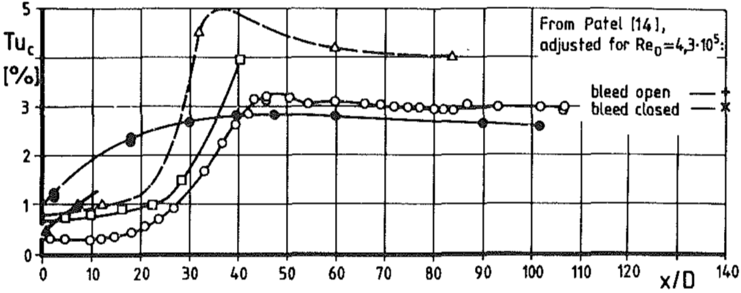
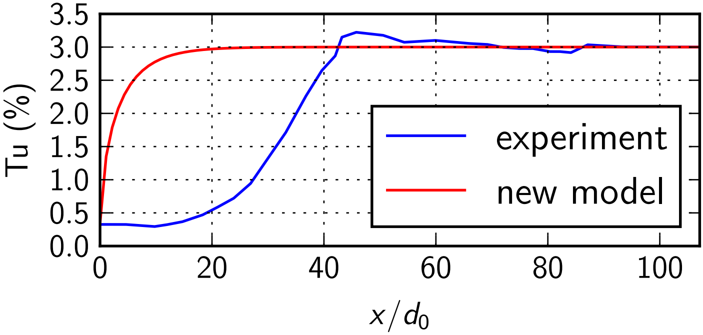


Figure: Compilation of developing flow data from Klein (1981).

$$\overline{\mathrm{Tu}}_0^2 = \overline{\mathrm{Tu}}_{\mathrm{FD}}^2 + \left(\overline{\mathrm{Tu}}_{\mathrm{c}}^2 - \overline{\mathrm{Tu}}_{\mathrm{FD}}^2\right) \exp\left(-\frac{3\overline{\mathrm{Tu}}_{\mathrm{FD}}^2}{f}\frac{L_0}{d_0}\right)$$

Nozzle turbulence model (4/4)



$$\overline{\mathrm{Tu}}_0^2 = \overline{\mathrm{Tu}}_{\mathrm{FD}}^2 + \left(\overline{\mathrm{Tu}}_{\mathrm{c}}^2 - \overline{\mathrm{Tu}}_{\mathrm{FD}}^2\right) \exp\left(-\frac{3\overline{\mathrm{Tu}}_{\mathrm{FD}}^2}{f}\frac{L_0}{d_0}\right)$$

Summary

- The nozzle turbulence model developed by Huh and Gosman is inaccurate.
- The basic idea behind their model is valid, however: Turbulence quantities are related to friction factors and loss coefficients.
- This can be done through the *turbulent Bernoulli equation*.
- Turbulence modeling is still required.
- A replacement for the Huh's nozzle turbulence model has been developed, but this model requires improvement and better experimental validation.

Questions?

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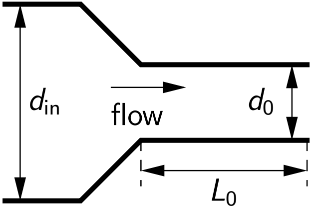
Paper:

Estimating k and ε with loss coefficients
Preprint DOI: 10.31224/OSF.IO/QSFP7

Background: Huh's model for nozzle turbulence (extra)

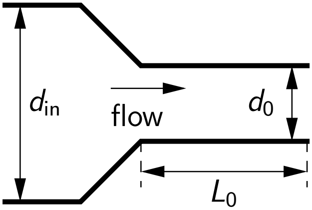


$$\overline{k}_0 = \frac{\overline{U}_0^2}{8L_0/d_0}\left[\frac{1}{C_{\text{d}}^2} - \zeta_{\text{c}} - \left(1 - c^{-2}\right)\right]$$

Sanity check: Does the proposed model have the correct sign?

Yes, if $\zeta_{\text{c}} < \frac{1}{C_{\text{d}}^2} - 1$ for large c. (Not necessarily satisfied.)

Background: Huh's model for nozzle turbulence (extra)

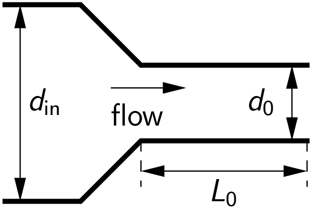


$$\overline{k}_0 = \frac{\overline{U}_0^2}{8L_0/d_0}\left[\frac{1}{C_{\text{d}}^2} - \zeta_{\text{c}} - \left(1 - c^{-2}\right)\right]$$

Sanity check: Does the proposed model have consistent dimensions?

Yes.

Background: Huh's model for nozzle turbulence (extra)



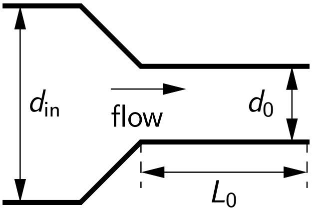
$$\overline{k}_0 = \frac{\overline{U}_0^2}{8L_0/d_0}\left[\frac{1}{C_{\text{d}}^2} - \zeta_{\text{c}} - \left(1 - c^{-2}\right)\right]$$

Sanity check: Does the proposed model have the expected functional dependencies?

No. I expect the nozzle inlet turbulent kinetic energy ($\overline{k}_{\text{in}}$) to be a factor, but it's absent.

The importance of the inlet turbulent kinetic energy has been shown experimentally by Klein (1981), Ervine, E. McKeogh, and Elsawy (1980), and E. J. McKeogh and Elsawy (1980).

Background: Huh's model for nozzle turbulence (extra)

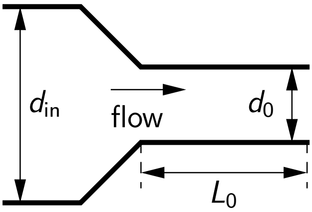


$$\overline{k}_0 = \frac{\overline{U}_0^2}{8L_0/d_0}\left[\frac{1}{C_d^2} - \zeta_c - \left(1 - c^{-2}\right)\right]$$

Sanity check: Does the proposed model have the expected trend?

No. Intuitively, one expects that the turbulent kinetic energy would increase as the nozzle becomes longer, but the model predicts the exact opposite trends. (This assumes that the inlet turbulent kinetic energy is small compared against the fully developed value.)

Background: Huh's model for nozzle turbulence (extra)



$$\overline{k}_0 = \frac{\overline{U}_0^2}{8 L_0/d_0} \left[\frac{1}{C_\mathrm{d}^2} - \zeta_\mathrm{c} - \left(1 - c^{-2}\right) \right]$$

Sanity check: Does the proposed model have the correct behavior in the limits?

No. As the nozzle becomes very long ($L_0/d_0 \to \infty$) I expect the turbulent kinetic energy to go to the fully developed value, $\overline{k}_\mathrm{FD}$. But instead it goes to zero.

Worse, as $L_0/d_0 \downarrow 0$, $\overline{k}_0$ increases to infinity.

References I

Ervine, D. A., E. McKeogh, and E. M. Elsawy (1980). "Effect of Turbulence Intensity on the Rate of Air Entrainment by Plunging Water Jets". In: *Proceedings of the Institution of Civil Engineers* 69.2, pp. 425–445.

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